| 6.1	**Exercises**	The *HM mathSpace®* CD-ROM and *Eduspace®* for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

VOCABULARY CHECK: Fill in the blanks.

1. An _____ triangle is a triangle that has no right angle.

2. For triangle ABC, the Law of Sines is given by $\dfrac{a}{\sin A} =$ _____ $= \dfrac{c}{\sin C}$.

3. The area of an oblique triangle is given by $\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C =$ _____ .

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–18, use the Law of Sines to solve the triangle. Round your answers to two decimal places.

1.

2.

3.

4.
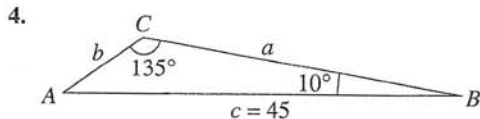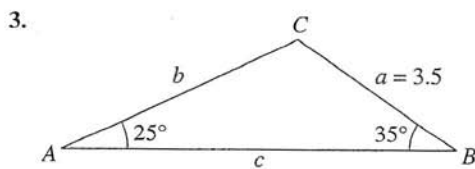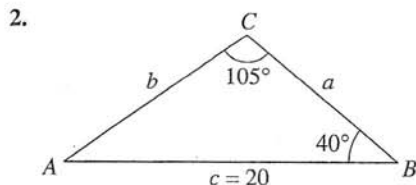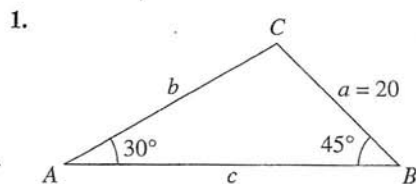
5. $A = 36°,\quad a = 8,\quad b = 5$
6. $A = 60°,\quad a = 9,\quad c = 10$
7. $A = 102.4°,\quad C = 16.7°,\quad a = 21.6$
8. $A = 24.3°,\quad C = 54.6°,\quad c = 2.68$
9. $A = 83°\,20',\quad C = 54.6°,\quad c = 18.1$
10. $A = 5°\,40',\quad B = 8°\,15',\quad b = 4.8$
11. $B = 15°\,30',\quad a = 4.5,\quad b = 6.8$
12. $B = 2°\,45',\quad b = 6.2,\quad c = 5.8$
13. $C = 145°,\quad b = 4,\quad c = 14$

14. $A = 100°,\quad a = 125,\quad c = 10$
15. $A = 110°\,15',\quad a = 48,\quad b = 16$
16. $C = 85°\,20',\quad a = 35,\quad c = 50$
17. $A = 55°,\quad B = 42°,\quad c = \frac{3}{4}$
18. $B = 28°,\quad C = 104°,\quad a = 3\frac{5}{8}$

In Exercises 19–24, use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

19. $A = 110°,\quad a = 125,\quad b = 100$
20. $A = 110°,\quad a = 125,\quad b = 200$
21. $A = 76°,\quad a = 18,\quad b = 20$
22. $A = 76°,\quad a = 34,\quad b = 21$
23. $A = 58°,\quad a = 11.4,\quad b = 12.8$
24. $A = 58°,\quad a = 4.5,\quad b = 12.8$

In Exercises 25–28, find values for b such that the triangle has (a) one solution, (b) two solutions, and (c) no solution.

25. $A = 36°,\quad a = 5$
26. $A = 60°,\quad a = 10$
27. $A = 10°,\quad a = 10.8$
28. $A = 88°,\quad a = 315.6$

In Exercises 29–34, find the area of the triangle having the indicated angle and sides.

29. $C = 120°,\quad a = 4,\quad b = 6$
30. $B = 130°,\quad a = 62,\quad c = 20$
31. $A = 43°\,45',\quad b = 57,\quad c = 85$
32. $A = 5°\,15',\quad b = 4.5,\quad c = 22$
33. $B = 72°\,30',\quad a = 105,\quad c = 64$
34. $C = 84°\,30',\quad a = 16,\quad b = 20$

35. Height · Because of prevailing winds, a tree grew so that it was leaning 4° from the vertical. At a point 35 meters from the tree, the angle of elevation to the top of the tree is 23° (see figure). Find the height h of the tree.
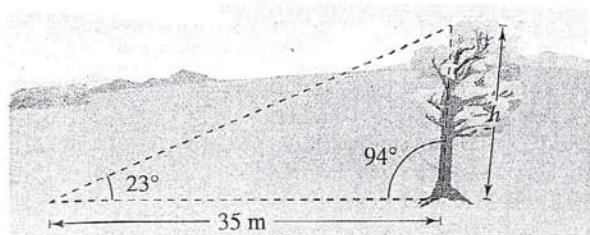

36. Height A flagpole at a right angle to the horizontal is located on a slope that makes an angle of 12° with the horizontal. The flagpole's shadow is 16 meters long and points directly up the slope. The angle of elevation from the tip of the shadow to the sun is 20°.

(a) Draw a triangle that represents the problem. Show the known quantities on the triangle and use a variable to indicate the height of the flagpole.

(b) Write an equation involving the unknown quantity.

(c) Find the height of the flagpole.

37. Angle of Elevation A 10-meter telephone pole casts a 17-meter shadow directly down a slope when the angle of elevation of the sun is 42° (see figure). Find θ, the angle of elevation of the ground.
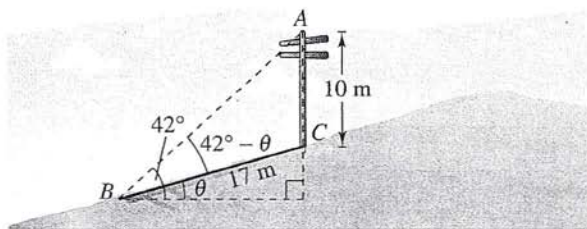
38. Flight Path A plane flies 500 kilometers with a bearing of 316° from Naples to Elgin (see figure). The plane then flies 720 kilometers from Elgin to Canton. Find the bearing of the flight from Elgin to Canton.
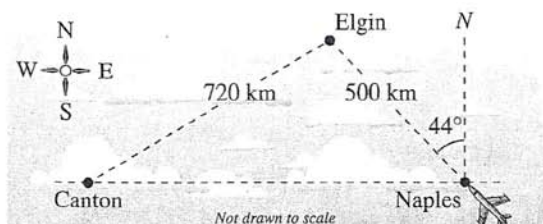
39. Bridge Design A bridge is to be built across a small lake from a gazebo to a dock (see figure). The bearing from the gazebo to the dock is S 41° W. From a tree 100 meters from the gazebo, the bearings to the gazebo and the dock are S 74° E and S 28° E, respectively. Find the distance from the gazebo to the dock.
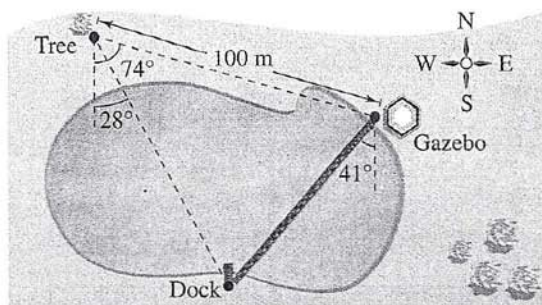

40. Railroad Track Design The circular arc of a railroad curve has a chord of length 3000 feet and a central angle of 40°.

(a) Draw a diagram that visually represents the problem. Show the known quantities on the diagram and use the variables r and s to represent the radius of the arc and the length of the arc, respectively.

(b) Find the radius r of the circular arc.

(c) Find the length s of the circular arc.

41. Glide Path A pilot has just started on the glide path for landing at an airport with a runway of length 9000 feet. The angles of depression from the plane to the ends of the runway are 17.5° and 18.8°.

(a) Draw a diagram that visually represents the problem.

(b) Find the air distance the plane must travel until touching down on the near end of the runway.

(c) Find the ground distance the plane must travel until touching down.

(d) Find the altitude of the plane when the pilot begins the descent.

42. Locating a Fire The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 70° E from Colt Station (see figure). Find the distance of the fire from each tower.
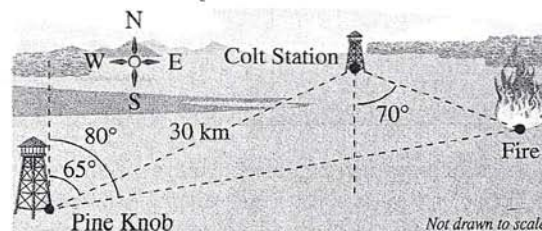
43. *Distance* A boat is sailing due east parallel to the shoreline at a speed of 10 miles per hour. At a given time, the bearing to the lighthouse is S 70° E, and 15 minutes later the bearing is S 63° E (see figure). The lighthouse is located at the shoreline. What is the distance from the boat to the shoreline?
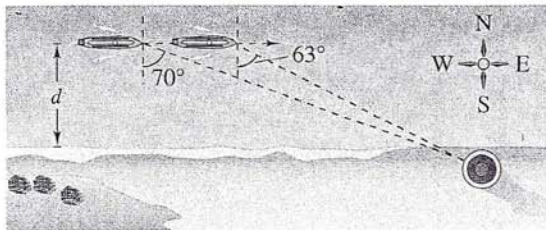
Model It

44. *Shadow Length* The Leaning Tower of Pisa in Italy is characterized by its tilt. The tower leans because it was built on a layer of unstable soil—clay, sand, and water. The tower is approximately 58.36 meters tall from its foundation (see figure). The top of the tower leans about 5.45 meters off center.
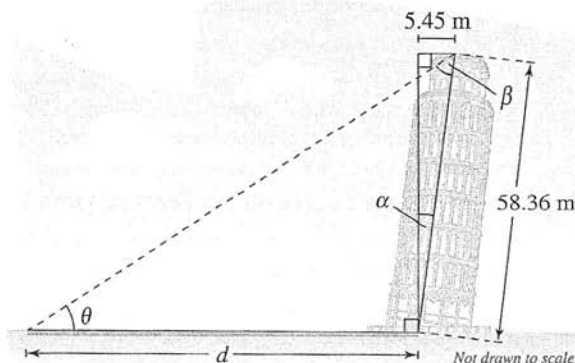
(a) Find the angle of lean α of the tower.

(b) Write β as a function of d and θ, where θ is the angle of elevation to the sun.

(c) Use the Law of Sines to write an equation for the length d of the shadow cast by the tower.

(d) Use a graphing utility to complete the table.

θ	10°	20°	30°	40°	50°	60°
d						

Synthesis

True or False? In Exercises 45 and 46, determine whether the statement is true or false. Justify your answer.

45. If a triangle contains an obtuse angle, then it must be oblique.

46. Two angles and one side of a triangle do not necessarily determine a unique triangle.

47. *Graphical and Numerical Analysis* In the figure, α and β are positive angles.

(a) Write α as a function of β.

(b) Use a graphing utility to graph the function. Determine its domain and range.

(c) Use the result of part (a) to write c as a function of β.

(d) Use a graphing utility to graph the function in part (c). Determine its domain and range.

(e) Complete the table. What can you infer?

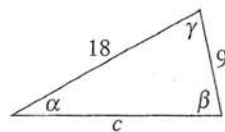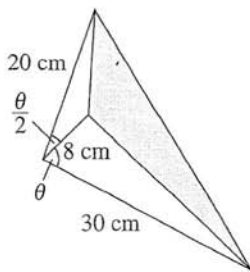| β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α							
c							

FIGURE FOR 47   FIGURE FOR 48

48. *Graphical Analysis*

(a) Write the area A of the shaded region in the figure as a function of θ.

(b) Use a graphing utility to graph the area function.

(c) Determine the domain of the area function. Explain how the area of the region and the domain of the function would change if the eight-centimeter line segment were decreased in length.

Skills Review

In Exercises 49–52, use the fundamental trigonometric identities to simplify the expression.

49. $\sin x \cot x$

50. $\tan x \cos x \sec x$

51. $1 - \sin^2\left(\dfrac{\pi}{2} - x\right)$

52. $1 + \cot^2\left(\dfrac{\pi}{2} - x\right)$

6.2 | Exercises

VOCABULARY CHECK: Fill in the blanks.

1. If you are given three sides of a triangle, you would use the Law of _____ to find the three angles of the triangle.

2. The standard form of the Law of Cosines for $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ is _____ .

3. The Law of Cosines can be used to establish a formula for finding the area of a triangle called _____ _____ Formula.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–16, use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

1.

$b = 10$, C, $a = 7$, A, $c = 15$, B

2.

$b = 3$, C, $a = 8$, A, $c = 9$, B

3.

$b = 15$, C, a, $30°$, A, $c = 30$, B

4.

$b = 4.5$, C, $a = 10$, $105°$, A, c, B

5. $a = 11$, $b = 14$, $c = 20$

6. $a = 55$, $b = 25$, $c = 72$

7. $a = 75.4$, $b = 52$, $c = 52$

8. $a = 1.42$, $b = 0.75$, $c = 1.25$

9. $A = 135°$, $b = 4$, $c = 9$

10. $A = 55°$, $b = 3$, $c = 10$

11. $B = 10° \, 35'$, $a = 40$, $c = 30$

12. $B = 75° \, 20'$, $a = 6.2$, $c = 9.5$

13. $B = 125° 40'$, $a = 32$, $c = 32$

14. $C = 15° \, 15'$, $a = 6.25$, $b = 2.15$

15. $C = 43°$, $a = \frac{4}{9}$, $b = \frac{7}{9}$

16. $C = 103°$, $a = \frac{3}{8}$, $b = \frac{3}{4}$

In Exercises 17–22, complete the table by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by c and d.)
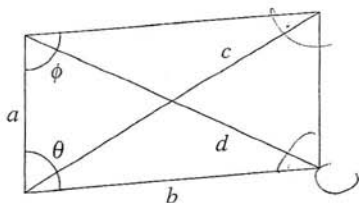
|	a	b	c	d	θ	ϕ
17.	5	8			45°	
18.	25	35				120°
19.	10	14	20			
20.	40	60		80		
21.	15		25	20		
22.		25	50	35		

In Exercises 23–28, use Heron's Area Formula to find the area of the triangle.

23. $a = 5$, $b = 7$, $c = 10$

24. $a = 12$, $b = 15$, $c = 9$

25. $a = 2.5$, $b = 10.2$, $c = 9$

26. $a = 75.4$, $b = 52$, $c = 52$

27. $a = 12.32$, $b = 8.46$, $c = 15.05$

28. $a = 3.05$, $b = 0.75$, $c = 2.45$

29. *Navigation* A boat race runs along a triangular course marked by buoys A, B, and C. The race starts with the boats headed west for 3700 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1700 meters and 3000 meters. Draw a figure that gives a visual representation of the problem, and find the bearings for the last two legs of the race.

30. *Navigation* A plane flies 810 miles from Franklin to Centerville with a bearing of 75°. Then it flies 648 miles from Centerville to Rosemount with a bearing of 32°. Draw a figure that visually represents the problem, and find the straight-line distance and bearing from Franklin to Rosemount.

31. *Surveying* To approximate the length of a marsh, a surveyor walks 250 meters from point A to point B, then turns $75°$ and walks 220 meters to point C (see figure). Approximate the length AC of the marsh.
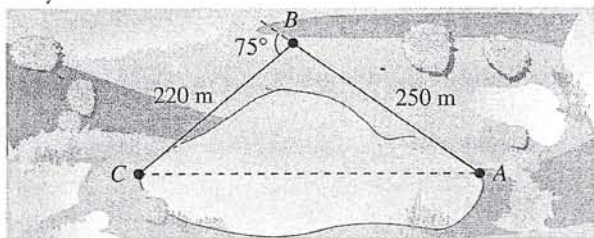

32. *Surveying* A triangular parcel of land has 115 meters of frontage, and the other boundaries have lengths of 76 meters and 92 meters. What angles does the frontage make with the two other boundaries?

33. *Surveying* A triangular parcel of ground has sides of lengths 725 feet, 650 feet, and 575 feet. Find the measure of the largest angle.

**34. *Streetlight Design*** Determine the angle θ in the design of the streetlight shown in the figure.
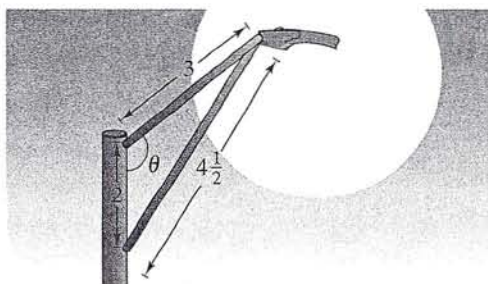
35. *Distance* Two ships leave a port at 9 A.M. One travels at a bearing of N $53°$ W at 12 miles per hour, and the other travels at a bearing of S $67°$ W at 16 miles per hour. Approximate how far apart they are at noon that day.

36. *Length* A 100-foot vertical tower is to be erected on the side of a hill that makes a $6°$ angle with the horizontal (see figure). Find the length of each of the two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.
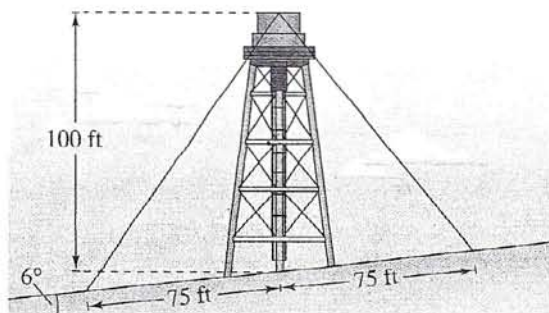

37. *Navigation* On a map, Orlando is 178 millimeters due south of Niagara Falls, Denver is 273 millimeters from Orlando, and Denver is 235 millimeters from Niagara Falls (see figure).
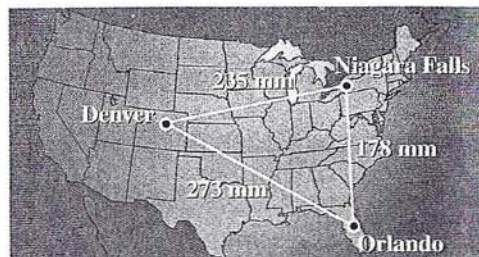
(a) Find the bearing of Denver from Orlando.

(b) Find the bearing of Denver from Niagara Falls.

38. *Navigation* On a map, Minneapolis is 165 millimeters due west of Albany, Phoenix is 216 millimeters from Minneapolis, and Phoenix is 368 millimeters from Albany (see figure).
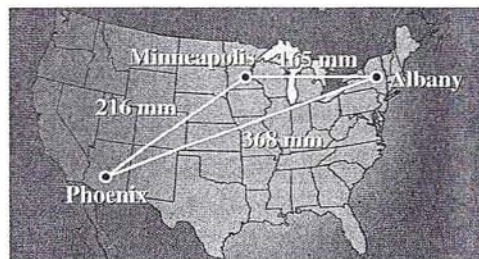
(a) Find the bearing of Minneapolis from Phoenix.

(b) Find the bearing of Albany from Phoenix.

39. *Baseball* On a baseball diamond with 90-foot sides, the pitcher's mound is 60.5 feet from home plate. How far is it from the pitcher's mound to third base?

40. *Baseball* The baseball player in center field is playing approximately 330 feet from the television camera that is behind home plate. A batter hits a fly ball that goes to the wall 420 feet from the camera (see figure). The camera turns $8°$ to follow the play. Approximately how far does the center fielder have to run to make the catch?
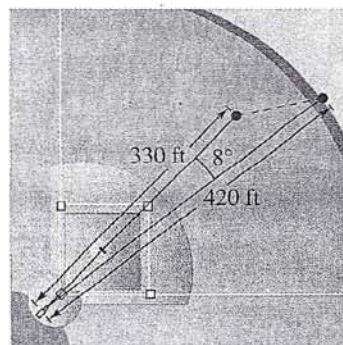
41. Aircraft Tracking To determine the distance between two aircraft, a tracking station continuously determines the distance to each aircraft and the angle A between them (see figure). Determine the distance a between the planes when $A = 42°$, $b = 35$ miles, and $c = 20$ miles.
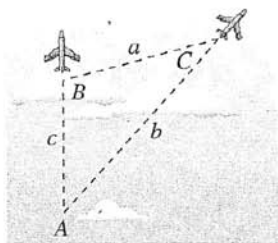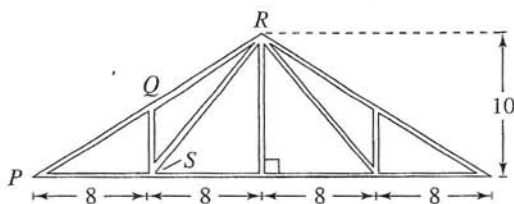

FIGURE FOR 41

42. Aircraft Tracking Use the figure for Exercise 41 to determine the distance a between the planes when $A = 11°$, $b = 20$ miles, and $c = 20$ miles.

43. Trusses Q is the midpoint of the line segment $\overline{PR}$ in the truss rafter shown in the figure. What are the lengths of the line segments $\overline{PQ}$, $\overline{QS}$, and $\overline{RS}$?


Model It

**44. Engine Design** An engine has a seven-inch connecting rod fastened to a crank (see figure).
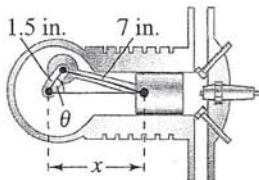
(a) Use the Law of Cosines to write an equation giving the relationship between x and θ.

(b) Write x as a function of θ. (Select the sign that yields positive values of x.)

(c) Use a graphing utility to graph the function in part (b).

(d) Use the graph in part (c) to determine the maximum distance the piston moves in one cycle.

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45. Paper Manufacturing In a process with continuous paper, the paper passes across three rollers of radii 3 inches, 4 inches, and 6 inches (see figure). The centers of the three-inch and six-inch rollers are d inches apart, and the length of the arc in contact with the paper on the four-inch roller is s inches. Complete the table.
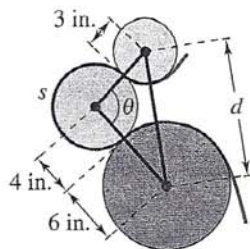

d (inches)	9	10	12	13	14	15	16
θ (degrees)							
s (inches)							

46. Awning Design A retractable awning above a patio door lowers at an angle of 50° from the exterior wall at a height of 10 feet above the ground (see figure). No direct sunlight is to enter the door when the angle of elevation of the sun is greater than 70°. What is the length x of the awning?
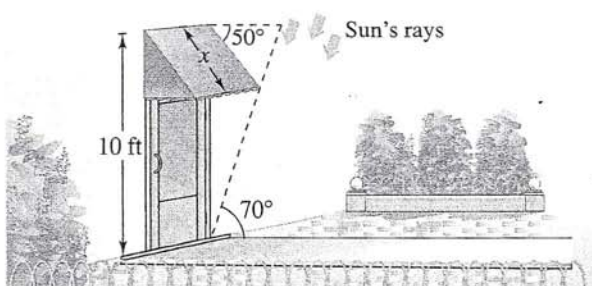
47. Geometry The lengths of the sides of a triangular parcel of land are approximately 200 feet, 500 feet, and 600 feet. Approximate the area of the parcel.

48. Geometry A parking lot has the shape of a parallelogram (see figure). The lengths of two adjacent sides are 70 meters and 100 meters. The angle between the two sides is 70°. What is the area of the parking lot?
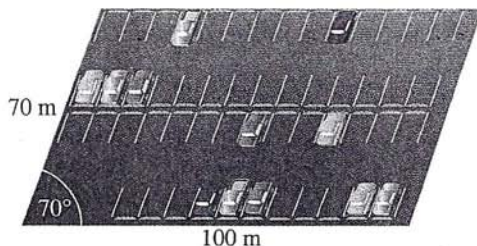
49. *Geometry* You want to buy a triangular lot measuring 510 yards by 840 yards by 1120 yards. The price of the land is $2000 per acre. How much does the land cost? (*Hint:* 1 acre = 4840 square yards)

50. *Geometry* You want to buy a triangular lot measuring 1350 feet by 1860 feet by 2490 feet. The price of the land is $2200 per acre. How much does the land cost? (*Hint:* 1 acre = 43,560 square feet)

Synthesis

True or False? In Exercises 51–53, determine whether the statement is true or false. Justify your answer.

51. In Heron's Area Formula, s is the average of the lengths of the three sides of the triangle.

52. In addition to SSS and SAS, the Law of Cosines can be used to solve triangles with SSA conditions.

53. A triangle with side lengths of 10 centimeters, 16 centimeters, and 5 centimeters can be solved using the Law of Cosines.

54. *Circumscribed and Inscribed Circles* Let R and r be the radii of the circumscribed and inscribed circles of a triangle ABC, respectively (see figure), and let

$$s = \frac{a+b+c}{2}.$$
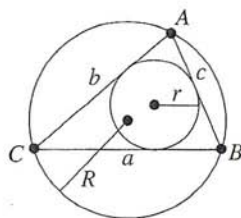
(a) Prove that $2R = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}.$

(b) Prove that $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}.$

Circumscribed and Inscribed Circles In Exercises 55 and 56, use the results of Exercise 54.

55. Given a triangle with

$a = 25, b = 55,$ and $c = 72$

find the areas of (a) the triangle, (b) the circumscribed circle, and (c) the inscribed circle.

56. Find the length of the largest circular running track that can be built on a triangular piece of property with sides of lengths 200 feet, 250 feet, and 325 feet.

57. *Proof* Use the Law of Cosines to prove that

$$\frac{1}{2}bc(1+\cos A) = \frac{a+b+c}{2} \cdot \frac{-a+b+c}{2}.$$

58. *Proof* Use the Law of Cosines to prove that

$$\frac{1}{2}bc(1-\cos A) = \frac{a-b+c}{2} \cdot \frac{a+b-c}{2}.$$

Skills Review

In Exercises 59–64, evaluate the expression without using a calculator.

59. $\arcsin(-1)$

60. $\arccos 0$

61. $\arctan \sqrt{3}$

62. $\arctan\left(-\sqrt{3}\right)$

63. $\arcsin\left(-\dfrac{\sqrt{3}}{2}\right)$

64. $\arccos\left(-\dfrac{\sqrt{3}}{2}\right)$

In Exercises 65–68, write an algebraic expression that is equivalent to the expression.

65. $\sec(\arcsin 2x)$

66. $\tan(\arccos 3x)$

67. $\cot[\arctan(x-2)]$

68. $\cos\left(\arcsin \dfrac{x-1}{2}\right)$

In Exercises 69–72, use trigonometric substitution to write the algebraic equation as a trigonometric function of θ, where $-\pi/2 < \theta < \pi/2$. Then find $\sec\theta$ and $\csc\theta$.

69. $5 = \sqrt{25 - x^2}, \quad x = 5\sin\theta$

70. $-\sqrt{2} = \sqrt{4 - x^2}, \quad x = 2\cos\theta$

71. $-\sqrt{3} = \sqrt{x^2 - 9}, \quad x = 3\sec\theta$

72. $12 = \sqrt{36 + x^2}, \quad x = 6\tan\theta$

In Exercises 73 and 74, write the sum or difference as a product.

73. $\cos\dfrac{5\pi}{6} - \cos\dfrac{\pi}{3}$

74. $\sin\left(x - \dfrac{\pi}{2}\right) - \sin\left(x + \dfrac{\pi}{2}\right)$